Fractional Burgers equation with singular initial condition

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1. Introduction

Let $d \geq 2$ and $\alpha \in (1,2)$. Consider the fractional Burgers equation

$$\begin{cases} u_t = \Delta^{\alpha/2} u + b \cdot \nabla(u|u|^q), & t > 0, \\ u(0,\cdot) = u_0 \end{cases}$$

in $\mathbb{R}^d$, where $\beta > 1$, $b \in \mathbb{R}^d$ and $q = \frac{\alpha-1}{\beta}$ are fixed. Here, $\Delta^{\alpha/2}$ is the fractional Laplacian.

This equation was studied by several authors (see e.g. [1], [2], [3]). In general, the standing assumption in these papers was $u_0 \in L^1(\mathbb{R}^d)$. We will present the results of the paper [4] in which we introduced a new class of initial conditions. Our main example is $u_0(x) = c|x|^{-\beta}$, which does not belong to any L^p, $p \geq 1$. In the talk we will focus on the existence of solutions $u(t,x)$ and their properties.

References

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[3] T. Jakubowski, G. Serafin, *Stable estimates for source solution of critical fractal Burgers equation*, Nonlinear Anal. 130 (2016), 396407.

[4] T. Jakubowski, G. Serafin, *Fractional Burgers equation with singular initial condition*, Nonlinear Anal. 232 (2023), 113269.