Density results and trace operator in weighted Sobolev spaces defined on the half-line, equipped with power weights

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During the lecture, we will study the properties of $W_0^{1,p}(\mathbb{R}_+, t^\beta)$ - the completion of $C_0^\infty(\mathbb{R}_+)$ in the power-weighted Sobolev spaces $W^{1,p}(\mathbb{R}_+, t^\beta)$, where $\beta \in \mathbb{R}$. Among other results, the analytic characterization of $W_0^{1,p}(\mathbb{R}_+, t^\beta)$ for all $\beta \in \mathbb{R}$ will be presented. Our analysis is based on the precise study of the two trace operators: $Tr^0(u) := \lim_{t \to 0} u(t)$ and $Tr^\infty(u) := \lim_{t \to \infty} u(t)$, which leads to the analysis of the asymptotic behavior of functions from $W_0^{1,p}(\mathbb{R}_+, t^\beta)$ near zero or infinity. The obtained statements can contribute to the proper formulation of Boundary Value Problems in ODEs, or PDEs with radial symmetries. One can also apply our results to some questions in the complex interpolation theory, raised by M. Cwikel and A. Einav in 2019, which we discuss within the particular case of Sobolev spaces $W^{1,p}(\mathbb{R}_+, t^\beta)$.