# INVERSION PROBLEM IN MEASURE AND FOURIER-STIELTJES ALGEBRAS

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Let $G$ be a locally compact Abelian group with its dual $\widehat{G}$ and let $M(G)$ denote the Banach algebra of complex-valued measures on $G$. The classical Wiener-Pitt phenomenon asserts that the spectrum of a measure may be strictly larger than the closure of the range of its Fourier-Stieltjes transform. In particular, if $G$ is non-discrete, there exists $\mu \in M(G)$ such that $|\widehat{\mu}(\gamma)| > c > 0$ for every $\gamma \in \widehat{G}$ but $\mu$ is not invertible. In the paper [N], N. Nikolski suggested the following problem.

**Problem 1.** *Let $\mu \in M(G)$ satisfy $\|\mu\| \leq 1$ and $|\widehat{\mu}(\gamma)| \geq \delta$ for every $\gamma \in \widehat{G}$. What is the minimal value of $\delta_0$ assuring the invertibility of $\mu$ for every $\delta > \delta_0$? What can be said about the inverse (in terms of $\delta$)?*

In my talk I show that $\delta_0 = \frac{1}{2}$ is the optimal value for the first question (for non-discrete $G$). Also, I will present a partial solution for the quantitative variant of the problem (second question): if all elements of $G$ (except the unit) are of infinite order then we can control the norm of the inverse for every $\delta > \frac{-1+\sqrt{33}}{8} \simeq 0,593$. This improves the original result of Nikolski: $\delta > \frac{1}{\sqrt{2}} \simeq 0,707$.
If time permits I will present some generalizations of the aformentioned results for Fourier-Stieltjes algebras built on non-commutative groups.
The talk is based on a paper [OW] written in collaboration with Mateusz Wasilewski.

## References

[OW] P. Ohrysko, M. Wasilewski: *Inversion problem in measure and Fourier-Stieltjes algebras*, J. Funct. Anal. 278 (2020), no. 5, 108399, 19 pp.

[N] N. Nikolski: *In search of the invisible spectrum*, Ann. Inst. Fourier (Grenoble), vol. 49, no. 6, pp. 1925–1998, 1999.