Wiener–Luxemburg amalgam spaces

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In many cases of function spaces, the definition of the space imposes two distinct conditions on the functions: a local condition and a global one ("global" is to be understood in the sense of "near infinity"). In case of rearrangement invariant Banach function spaces the local condition prescribes how fast the blow-ups can be while the global one prescribes how fast the decay must be. As can be seen from the elementary example of Lebesgue spaces, those two conditions are independent and thus it is often useful or even necessary to prescribe them separately. To this end, we introduce the concept of the so-called Wiener–Luxemburg amalgam spaces, which provide an abstract framework for defining a space where the local and global behaviour is inherited, respectively, from a given pair of r.i. Banach function spaces.

We were inspired by the more classical concept of Wiener amalgam spaces (see [1]), which are a tool of similar nature. However, their construction is such that, when applied to a pair of r.i. Banach function spaces, the result is usually neither r.i. nor a Banach function space. This motivated our work to provide a tool that would be more compatible with this setting.

In this talk, we will present our construction and some selected key results, including the fact that the construction preserves the properties of r.i. Banach function spaces, a description of associate spaces, and some result concerning embeddings.

The talk is based on the paper [2], which also contains extension of the theory to the class of r.i. quasi-Banach function spaces and some applications.

References

[1] H. G. Feichtinger. Banach convolution algebras of Wiener type. In E. B. Sz.-Nagy and J. Szabados., editors, *Proc. Conf. on Functions, Series, Operators, Budapest 1980*, volume 35 of *Colloq. Math. Soc. Janos Bolyai*, pages 509–524. North-Holland, 1983.

[2] D. Peša. Wiener-Luxemburg amalgam spaces. *J. Funct. Anal.*, 282(1):Paper No. 109270, 47, 2022.