The Douglas formula in L^p

Artur Rutkowski

Wrocaw University of Science and Technology, Poland
E-mail: artur.rutkowski@pwr.edu.pl

The classical formula of Douglas [2] relates the Dirichlet energy of a harmonic function u on the unit disk $B(0,1) \subset \mathbb{R}^2$ to the energy of its boundary function g:

$$\int\limits_{B(0,1)} |\nabla u(x)|^2 \, dx = \frac{1}{8\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{(g(\eta) - g(\xi))^2}{\sin^2((\xi - \eta)/2)} \, d\eta \, d\xi. \tag{1}$$

The kernel $1/(\sin^2((\eta - \xi)/2))$ is the normal derivative of the Poisson kernel of the unit disk and is comparable to $|x - y|^2$, so the right-hand side is approximately the $H^{1/2}$ seminorm. This shows that (1) is strongly related to the trace theory for the Sobolev class $W^{1,2}$.

Our goal is to obtain an analogue of (1) in L^p, $p \in (1, \infty)$, for domains more general than the disk. Namely, we prove the following formula for $C^{1,1}$ domains D:

$$\int\limits_D |\nabla u(x)|^2 |u(x)|^{p-2} \, dx$$
$$= \frac{1}{2(p-1)} \int\limits_{\partial D} \int\limits_{\partial D} (g(z)^{\langle p-1 \rangle} - g(w)^{\langle p-1 \rangle})(g(z) - g(w)) \, \gamma_D(z, w) \, \sigma(dz) \, \sigma(dw).$$

Here, $a^{\langle p-1 \rangle} = a|a|^{p-2}$, σ is the surface measure on ∂D, and γ_D is the normal derivative of the Poisson kernel of D.

The talk is based on a joint work [1] with Krzysztof Bogdan and Damian Fafua (Wrocaw).

References

[1] K. Bogdan, D. Fafua, A. Rutkowski, The Douglas formula in L^p, *Nonlinear Differ. Equ. Appl. (NoDEA)*, **30**(2023), article number: 55.

[2] J. Douglas, Solution of the problem of Plateau, *Trans. Amer. Math. Soc.*, **33**(1931), 263–321.